Indian Statistical Institute

B. Math. Hons. I Year

Semestral Examination 2002-2003

Analysis II

Date:24-04-2003 Total Marks: 50 Instructor: T.S.S.R.K. Rao

1. Let f be a continuous function with compact support on R^n. Suppose $f \geq 0$ and $\int\limits_{R^n} f \, dx = 0$. Show that $f \equiv 0$. [5]

2. State and prove the change of variable theorem for a flip map on R^n. [5]

3. Let $U \subset R^n$ be an open set and $f : U \to R^m$ be a differentiable map such that $\|f'(x)\| \leq 1 \; \forall \; x \in U$. Show that f is uniformly continuous. [7]

4. Let C be a non-empty closed subset of R^n. Show that there is an $x_0 = (x_0(1), \ldots, x_0(n)) \in R^n$ such that

$$\left(\sum_{i=1}^{n} |x_0(i)|^2\right)^{1/2} = \inf_{y \in C} \left(\sum_{i=1}^{n} |y(i)|^2\right)^{1/2}.$$

[8]

5. Find and classify the exteme values defined by the function $f(x,y) = x^2 + y^2 + x + y + xy$. [6]

6. Let $Y \subset R^n$ and let $L : Y \to R$ be a uniformly continuous map. Show that there exists a $\tilde{L} : \bar{Y} \to R$ continuous, such that $\tilde{L} = L$ on Y. [8]

7. Let $A \subset R^n$ be a closed and bounded set and let $f : A \to \mathbb{C}$ be continuous. Show that f is uniformly continuous. [5]

8. Let $A \subset R^n$. Show that A with the metric induced from R^n is complete iff A is a closed set. [6]